1. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of k, giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

a) $u_1 = 2 \qquad u_2 = k - \frac{24}{u_1} \qquad u_3 = k - \frac{24}{u_2}$

$u_2 = k - \frac{24}{2} \qquad u_3 = k - \frac{24}{k-12}$ 1

$u_2 = k - 12$

$u_1 + 2u_2 + u_3 = 0$

sub in known values of u_1, u_2, u_3

$2 + 2(k-12) + k - \frac{24}{k-12} = 0$ 1

$3k - 22 - \frac{24}{k-12} = 0$

$\times (k-12)$

$(k-12)(3k-22) - 24 = 0$

$3k^2 - 22k - 36k + 264 - 24 = 0$

$3k^2 - 58k + 240 = 0$ 1 as required

b) we have $3k^2 - 58k + 240 = 0$

$(3k - 40)(k - 6) = 0$

$k = \frac{40}{3}$ or $k = 6$ (1)

choose $k = 6$ because k is an integer (1)

c) $U_3 = k - \frac{24}{k-12}$

$u_3 = 6 - \frac{24}{6-12}$

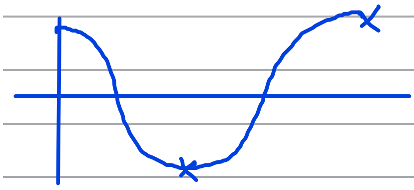
$u_3 = 6 + 4$

$u_3 = 10$ (1)

2.

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(3)



$$\cos(180n)^\circ = (-1)^n$$

$$\text{so} \quad \sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \sum_{n=2}^{\infty} \left(-\frac{3}{4}\right)^n$$

$$\left(\frac{3}{4}\right)^n (-1)^n = \left(-\frac{3}{4}\right)^n$$

geometric series with

$$a = \left(-\frac{3}{4}\right)^2 = \frac{9}{16} \quad (1)$$

$$r = -\frac{3}{4}$$

$$S_\infty = \frac{a}{1-r} = \frac{\frac{9}{16}}{1+\frac{3}{4}} = \frac{9}{28} \quad (1) \quad (1)$$

3. (a) Express $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \ln k$$

where k is a constant to be found.

(2)

(a)

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6:3} \frac{2}{x} \delta x = \int_{2.1}^{6.3} \frac{2}{x}\, dx \quad 1$$

(b)

$$\int_{2.1}^{6.3} \frac{2}{x}\, dx = \left[2 \ln x\right]_{2.1}^{6.3}$$

$$= (2 \ln 6.3) - (2 \ln 2.1) \quad 1$$

$$= 2 \ln \left(\frac{6.3}{2.1}\right)$$

$$= 2 \ln 3$$

$$= \ln 3^2$$

$$= \ln 9 \quad 1$$

$$\therefore k = 9$$

4. A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

(2)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2)

a) (i) $a_1 = 3$

$a_2 = 8 - 3 = 5$

$a_3 = 8 - 5 = 3$ (1)

$a_4 = 8 - 3 = 5$ (This sequence is periodic)

(ii) The order is 2 (1)

b) $\sum_{n=1}^{85} a_n = 3 + 5 + 3 + 5 + \ldots\ldots + 3$

43 x 3's = 129
(1)
42 x 5's = 210

Total = 129 + 210 = 339

$\therefore \sum_{n=1}^{85} a_n = 339$ (1)

5. A sequence $u_1, u_2, u_3 \ldots$ is defined by

$$u_1 = 35$$
$$u_{n+1} = u_n + 7\cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(a) (i) Show that $u_2 = 40$

(ii) Find the value of u_3 and the value of u_4

(3)

Given that the sequence is periodic with order 4

(b) (i) write down the value of u_5

(ii) find the value of $\sum_{r=1}^{25} u_r$

(3)

a) (i) $u_2 = 35 + 7\cos\left(\frac{\pi}{2}\right) - 5(-1)^1$

$= 40$ (1)

(ii) $u_3 = 40 + 7\cos\left(\frac{2\pi}{2}\right) - 5(-1)^2$

$= 28$ (1)

$u_4 = 28 + 7\cos\left(\frac{3\pi}{2}\right) - 5(-1)^3$

$= 33$ (1)

b) (i) $u_5 = u_1 = 35$ (1)

(ii) $\sum_{r=1}^{25} u_r = \sum_{r=1}^{24} u_r + u_{25}$

$= 6(35 + 40 + 28 + 33) + 35$ (1)

$= 851$ (1)